Computer Architecture and Operating Systems
Lecture 2: Data Representation

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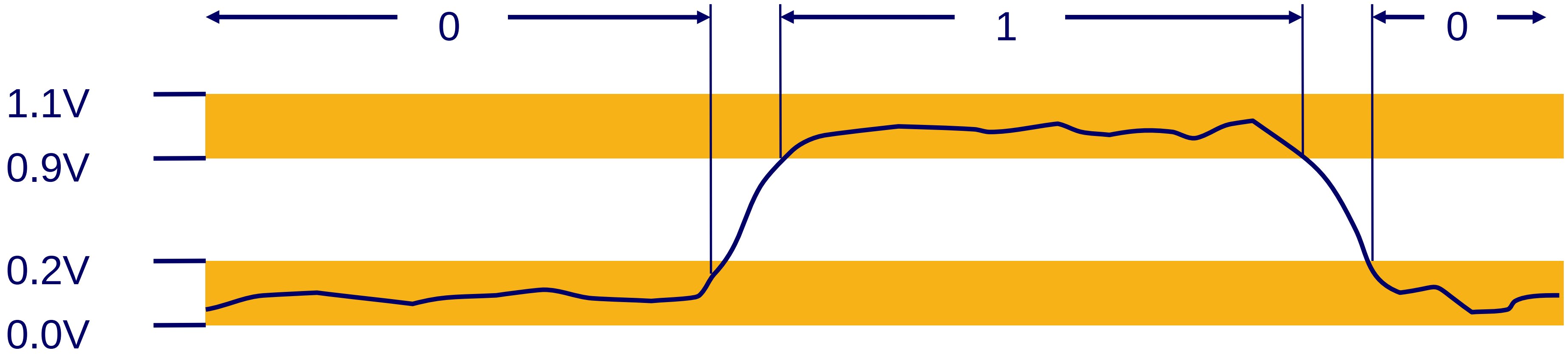
Everything is Bits

- Each bit is 0 or 1
- By encoding/interpreting sets of bits in various ways
  - Computers determine what to do (instructions)
  - … and represent and manipulate numbers, sets, strings, etc…
- Why bits? Electronic implementation
  - Easy to store with bistable elements
  - Reliably transmitted on noisy and inaccurate wires



Number Systems

- Decimal numbers

1000's column
100's column
10's column
1's column

$$5374_{10} = \underbrace{5 \times 10^3}_{\text{five thousands}} + \underbrace{3 \times 10^2}_{\text{three hundreds}} + \underbrace{7 \times 10^1}_{\text{seven tens}} + \underbrace{4 \times 10^0}_{\text{four ones}}$$

- Binary numbers

8's column
4's column
2's column
1's column

$$1101_2 = \underbrace{1 \times 2^3}_{\text{one eight}} + \underbrace{1 \times 2^2}_{\text{one four}} + \underbrace{0 \times 2^1}_{\text{no two}} + \underbrace{1 \times 2^0}_{\text{one one}} = 13_{10}$$

Powers of Two

- $2^0 = 1$
- $2^1 = 2$
- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$
- $2^5 = 32$
- $2^6 = 64$
- $2^7 = 128$
- Handy to memorize up to 2^{10}
- $2^8 = 256$
- $2^9 = 512$
- $2^{10} = 1024$
- $2^{11} = 2048$
- $2^{12} = 4096$
- $2^{13} = 8192$
- $2^{14} = 16384$
- $2^{15} = 32768$

Number Conversion

- Decimal to binary conversion:
  - Convert 10011_2 to decimal
  - $16\times1 + 8\times0 + 4\times0 + 2\times1 + 1\times1 = 19_{10}$

- Decimal to binary conversion:
  - Convert 47_{10} to binary
  - $32\times1 + 16\times0 + 8\times1 + 4\times1 + 2\times1 + 1\times1 = 101111_2$

Binary Values and Range

- *N*-digit decimal number
  - How many values? **10^N**
  - Range? **$[0, 10^N - 1]$**
  - Example: 3-digit decimal number:
    - **$10^3 = 1000$ possible values**
    - **Range: [0, 999]**
- *N*-bit binary number
  - How many values? **2^N**
  - Range: **$[0, 2^N - 1]$**
  - Example: 3-digit binary number:
    - **$2^3 = 8$ possible values**
    - **Range: $[0, 7] = [000_2$ to $111_2]$**

Hexadecimal Numbers

- Base 16
- Shorthand for binary

Hex Digit	Decimal Equivalent	Binary Equivalent
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
  - Convert $4AF_{16}$ (also written 0x4AF) to binary
  - $0100\ 1010\ 1111_2$
- Hexadecimal to decimal conversion:
  - Convert $4AF_{16}$ to decimal
  - $16^2 \times 4 + 16^1 \times 10 + 16^0 \times 15 = 1199_{10}$

Bits, Bytes, Nibbles…

- Bits

10010110

most significant bit

least significant bit

- Bytes & Nibbles

byte

10010110

nibble

- Bytes

CEBF9AD7

most significant byte

least significant byte

Encoding Byte Values

- Byte = 8 bits
  - Binary 00000000_2 to 11111111_2
  - Decimal: 0_{10} to 255_{10}
  - Hexadecimal 00_{16} to FF_{16}
    - Base 16 number representation
    - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
    - Write $FA1D37B_{16}$ in C as
      - 0xFA1D37B
      - 0xfa1d37b

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8
`float`	4	4
`double`	8	8
`long double`	−	−
pointer	4	8

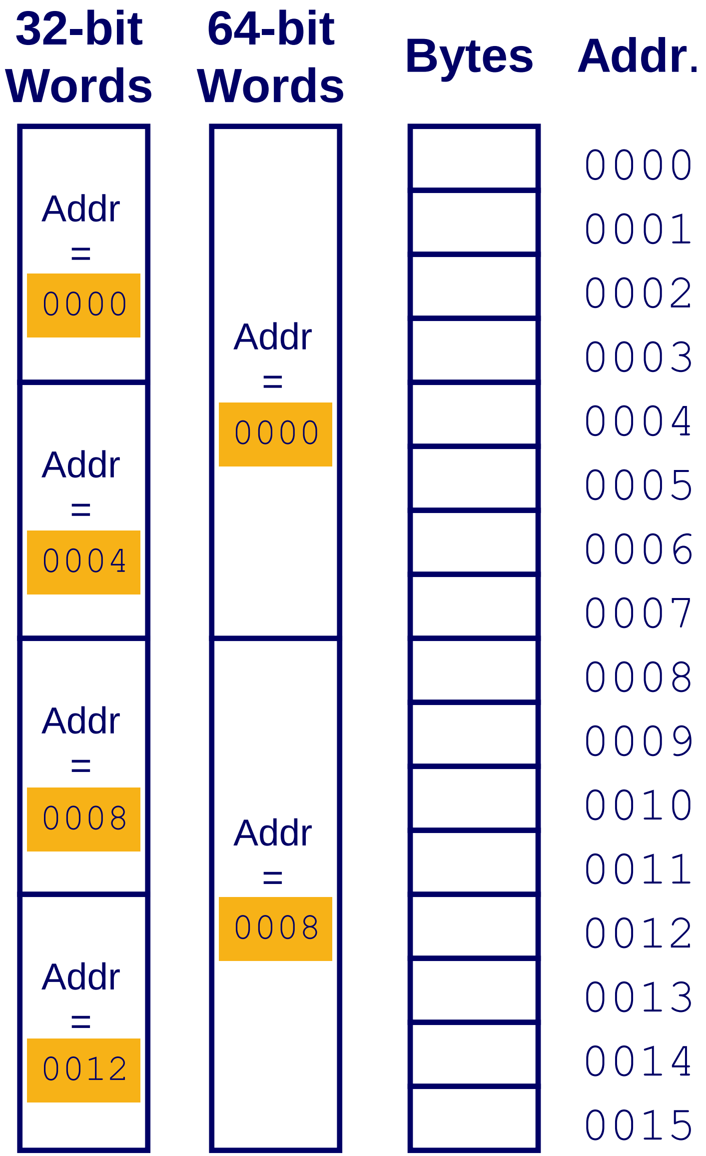
Byte-Oriented Memory Organization

- Programs refer to data by address
  - Conceptually, envision it as a very large array of bytes
    - In reality, it's not, but can think of it that way
  - An address is like an index into that array
    - and, a pointer variable stores an address
- Note: system provides private address spaces to each "process"
  - Think of a process as a program being executed
  - So, a program can clobber its own data, but not that of others

Machine Words

- Word is a native unit of information handled by computer
- Any computer has a "Word Size"
  - Nominal size of integer-valued data
    - and of addresses
  - Until recently, most machines used 32 bits (4 bytes) as word size
    - Limits addresses to 4GB (2^{32} bytes)
  - Increasingly, machines have 64-bit word size
    - Potentially, could have 18 EB (exabytes) of addressable memory
    - That's 18.4 X 10^{18}
  - Machines still support multiple data formats
    - Fractions or multiples of word size
    - Always integral number of bytes

Word-Oriented Memory Organization



- Addresses Specify Byte Locations
  - Address of first byte in word
  - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)

Byte Ordering

- How are the bytes within a multi-byte word ordered in memory?
- Conventions
  - **Big Endian**: Sun, PPC Mac, Internet
    - Least significant byte has highest address
  - **Little Endian**: x86, ARM processors running Android, iOS, and Windows, RISC-V
    - Least significant byte has lowest address

Byte Ordering Example

- Example
  - Variable x has 4-byte value of 0x01234567
  - Address given by &x is 0x100

Big Endian

		0x100	0x101	0x102	0x103		
		01	23	45	67		

Little Endian

		0x100	0x101	0x102	0x103		
		67	45	23	01		

Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

C short 2 bytes long

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
y	-15213	C4 93	11000100 10010011

Signed (two's complement)

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

- Sign Bit
  - For 2's complement, most significant bit indicates sign
    - 0 for nonnegative
    - 1 for negative

Two-complement Encoding Example

x = 15213: 00111011 01101101
y = -15213: 11000100 10010011

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		15213		-15213

Boolean Algebra

- Developed by George Boole in 19th Century
  - Algebraic representation of logic
    - Encode “True” as 1 and “False” as 0

And

- **A&B = 1 when both A=1 and B=1**

&	0	1
0	0	0
1	0	1

Or

- **A|B = 1 when either A=1 or B=1**

|	0	1
0	0	1
1	1	1

Not

- **~A = 1 when A=0**

~	
0	1
1	0

Exclusive-Or (Xor)

- **A^B = 1 when either A=1 or B=1, but not both**

^	0	1
0	0	1
1	1	0

Bitwise Operations

- Operate on Bit Vectors
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  --------     --------     --------     --------
  01000001     01111101     00111100     10101010
```

- All of the Properties of Boolean Algebra Apply

Logic Operations

- Operations &&, ||, !
  - Different from similar bitwise operations
  - View 0 as “False”
  - Anything nonzero as “True”
  - Always return 0 or 1
  - Early termination
- Examples (8-bit data type)
  - !0x41 => 0x00
  - !0x00 => 0x01
  - !!0x41 => 0x01
  - 0x69 && 0x55 => 0x01
  - 0x69 || 0x55 => 0x01

Sign-Extension

- Extend number from N to M bits (M > N)
- Sign bit is copied to most significant bits
- Number value is same
- Example 1:
  - 4-bit representation of 3 = 0011
  - 8-bit sign-extended value: 00000011
- Example 2:
  - 4-bit representation of -5 = 1011
  - 8-bit sign-extended value: 11111011

Zero-Extension

- Extend number from N to M bits (M > N)
- Zeros are copied to most significant bits
- Value changes for negative numbers
- Example 1:
  - 4-bit value = 0011 = 3_{10}
  - 8-bit zero-extended value: 00000011 = 3_{10}
- Example 2:
  - 4-bit value = 1011 = -5_{10}
  - 8-bit zero-extended value: 00001011 = 11_{10}

Shift Operations

- **Left Shift:** **x << y**
  - Shift bit-vector x left y positions
    - Throw away extra bits on left
  - Fill with 0's on right
- **Right Shift:** **x >> y**
  - Shift bit-vector x right y positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on left
- **Undefined Behavior**
  - Shift amount < 0 or ≥ word size

X	01100010
<< 3	00010000
Log. >> 2	00011000
Arith. >> 2	00011000

X	10100010
<< 3	00010000
Log. >> 2	00101000
Arith. >> 2	11101000

Integer Addition

- Decimal

$$\begin{array}{r} \color{steelblue}{11} \quad \leftarrow \textbf{Carries} \\ 3734 \\ +\ \underline{5168} \\ 8902 \end{array}$$

- Binary

$$\begin{array}{r} \color{steelblue}{11} \quad \leftarrow \textbf{Carries} \\ 1011 \\ +\ \underline{0011} \\ 1110 \end{array}$$

Integer Addition with Overflow

- Digital systems operate on a **fixed number of bits**
- Overflow: **result is too big** to fit in the bit size
- Example: 11 + 6 = 17 (or 1 if the bit size limit is 4):

$$
\begin{array}{r}
111 \\
1011 \\
+\ \underline{0110} \\
10001
\end{array}
$$

Integer Negation

- Negations means: complement and add 1
  - Complement means 1 → 0, 0 → 1

$$X + \sim X = 1111\ldots111_2 = -1$$
$$\sim X + 1 = -X$$

x = 0

	Decimal	Hex	Binary
0	0	00 00	00000000 00000000
~0	-1	FF FF	11111111 11111111
~0+1	0	00 00	00000000 00000000

x = 15213

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
~x	-15214	C4 92	11000100 10010010
~x+1	-15213	C4 93	11000100 10010011
y	-15213	C4 93	11000100 10010011

Integer Subtraction

- Add negation of second operand
- Example: 7 – 6 = 7 + (–6)

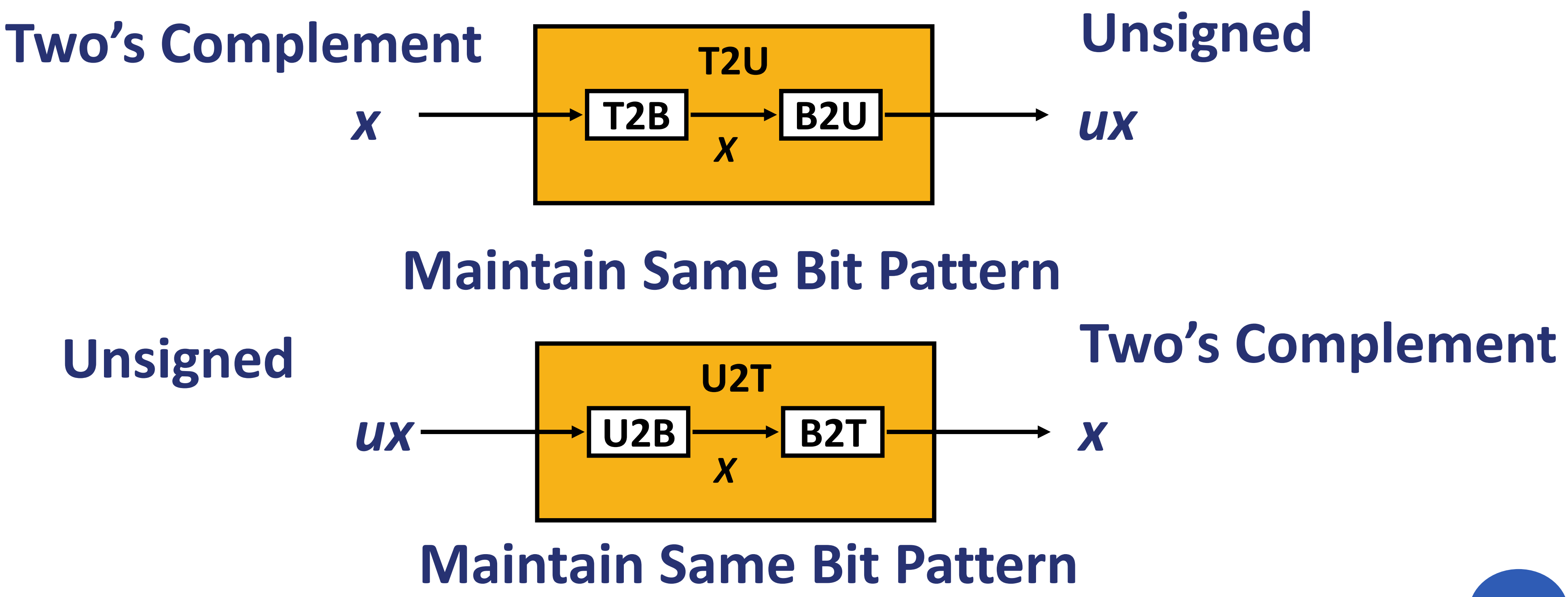
+7:	0000 0000 … 0000 0111
–6:	1111 1111 … 1111 1010
+1:	0000 0000 … 0000 0001

- Overflow if result out of range
  - Subtracting two +ve or two –ve operands, no overflow
  - Subtracting +ve from –ve operand
    - Overflow if result sign is 0
  - Subtracting –ve from +ve operand
    - Overflow if result sign is 1

Mapping Between Signed & Unsigned

- Mappings between unsigned and two's complement numbers:

Keep bit representations and reinterpret



Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

Signed ↔ Unsigned (0–7): =

Signed ↔ Unsigned (-8 to -1 ↔ 8–15): +/- 16

Any Questions?

```
            .text
__start:    addi t1, zero, 0x18
            addi t2, zero, 0x21
cycle:      beq t1, t2, done
            slt t0, t1, t2
            bne t0, zero, if_less
            nop
            sub t1, t1, t2
            j cycle
            nop
if_less:    sub t2, t2, t1
            j cycle
done:       add t3, t1, zero
```